Energy Conversion – Exam II

School of Engineering – Dep. of Industrial & Mechanical Eng.

Name: Solut° Key
Date: Tuesday, May 21st 2013; 06:00 PM
Location: ENG Attic
Instructor: Dr. Wassim Habchi
Notes: No documents allowed
Value: 25% of Total Grade
Time: 2 hours

[____ / 100]

Problem I (25 points)

Water is the working fluid in an ideal Rankine cycle. Superheated vapor enters the turbine at 8 MPa, 480°C. The condenser pressure is 8 kPa. The net power output of the cycle is 100 MW.

a) Sketch a T-s diagram for this cycle (**5 points**).

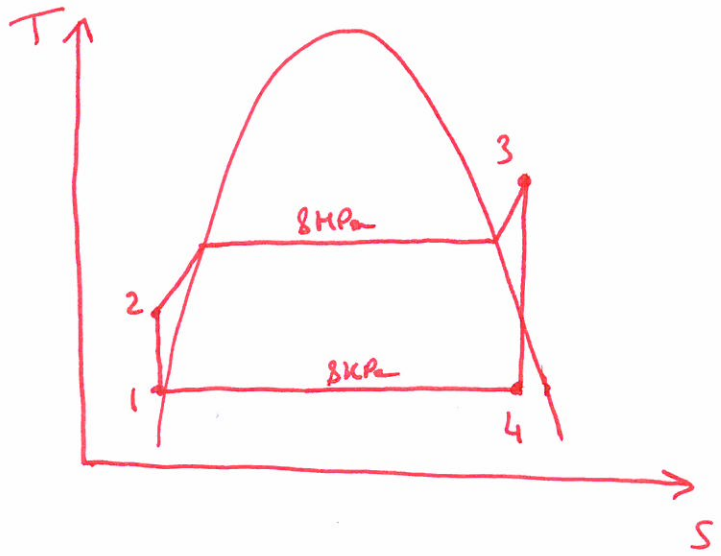
b) Determine the rate of heat transfer to the working fluid passing through the steam generator, in kW (**10 points**).

c) Determine the thermal efficiency of the cycle (**5 points**).

d) Determine the mass flow rate of condenser cooling water, in kg/h, if the cooling water enters the condenser at 15°C and exits at 35°C with negligible pressure change (**5 points**).

Solution:

a)



b) $\dot{Q}_H = ?$

$\dot{Q}_H = \dot{m}(h_3 - h_2)$

- $h_1 = h_f @ 8\text{kPa} = \boxed{173.362 \text{ kJ/kg}}$ $\quad v_1 = v_f @ 8\text{kPa} = 0.0010084 \text{ m}^3/\text{kg}$
- $h_2 \simeq h_1 + v_1(P_2 - P_1)$

$= 173.362 + 0.0010084 \times (8000 - 8) = \boxed{181.42 \text{ kJ/kg}}$

@③: $P_3 = 8\,MPa$, $T_3 = 480°C$ → $\boxed{h_3 = 3349.02\ KJ/Kg}$

$s_3 = 6.65912\ KJ/Kg.K$

@④: $P_4 = 8\,KPa$, $s_4 = s_3 = 6.65912\ KJ/Kg.K$ → $s_f < s_4 < s_g$

since $s_f = 0.59088\ KJ/Kg.K$

$s_g = 8.22984\ KJ/Kg.K$

$$\Rightarrow x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{6.65912 - 0.59088}{8.22984 - 0.59088} = 0.79438$$

$$\Rightarrow h_4 = h_f + x_4 h_{fg} = 173.362 + 0.79438 \times 2402.66 = \boxed{2081.987\ KJ/Kg}$$

$$\dot{W}_{net,out} = 100\ MW = \dot{m}(w_{turb} - w_{pump})$$
$$= \dot{m}(h_3 - h_4 - h_2 + h_1)$$

$$\Rightarrow \dot{m} = \frac{100 \times 10^3}{3349.02 - 2081.987 - 181.42 + 173.362} = 79.43\ Kg/s$$

$$\Rightarrow \dot{Q}_H = 79.43(3349.02 - 181.42) = \boxed{251602.468\ KW}$$

c) $$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{100 \times 10^3}{251602.468} = \boxed{0.39745 \simeq 39.75\%}$$

d) $$\dot{m}_w c_w \Delta T_w = \dot{Q}_L = \dot{m}(h_4 - h_1) = 79.43(2081.987 - 173.362)$$
$$= 151602.084\ KW$$

$$\dot{m}_w = \frac{151602.084}{c_w\ \Delta T_w} = \frac{151602.084}{4.18 \times (35-15)} = 1813.422\ Kg/s$$

(4.18 ↓ A-3)

$$= \boxed{6528319.396\ Kg/h}$$

$$= \boxed{6528.32\ Tons/h}$$

Problem II (25 points)

A vapor-compression refrigeration system circulates Refrigerant R-134a at a rate of 6 kg/min. The refrigerant enters the compressor at -10°C, 140 kPa, and exits at 700 kPa. The isentropic compressor efficiency is 67%. There are no appreciable pressure drops as the refrigerant flows through the condenser and evaporator. The refrigerant leaves the condenser at 700 kPa, ^as a sat. Liq.

a) Sketch a T-s diagram of this cycle (**5 points**).

b) Determine the coefficient of performance of this refrigerator (**10 points**).

c) Determine the refrigerating capacity, **in tons** and **in Btu/h** (**10 points**).

Solution:

@①: $T_1 = -10°C$, $P_1 = 140\,kPa$ → since $P < P_{sat}$ → Superheated Vapor

$h_1 = 246.36\ kJ/kg$

$s_1 = 0.9724\ kJ/kg.K$

@②s: $P_{2s} = 700\,kPa$, $s_{2s} = s_1 = 0.9724\ kJ/kg.K$ → $h_{2s} = 281.2\ kJ/kg$

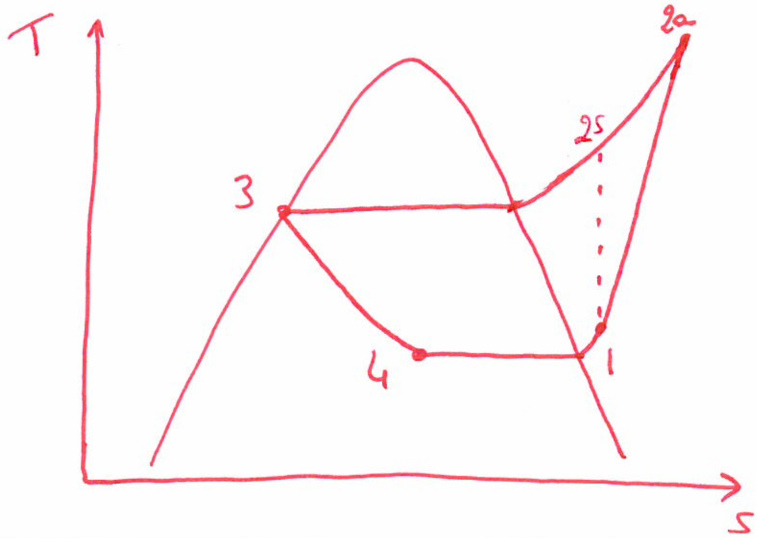
but $\eta_c = 0.67 = \frac{W_s}{W_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$

$\Rightarrow h_{2a} = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 246.36 + \frac{281.2 - 246.36}{0.67} = 298.36\ kJ/kg$

@③: $P_3 = 700\,kPa$, sat. Liq. → $h_3 = h_f @ 700\,kPa = 88.82\ kJ/kg$

$h_4 = h_3 = 88.82\ kJ/kg$

a)



b) $COP_R = \frac{q_L}{w_{in}} = \frac{h_1 - h_4}{h_{2a} - h_1} = \frac{246.36 - 88.82}{298.36 - 246.36}$

$\Rightarrow$ $\boxed{COP_R = 3.03}$

c) $\dot{Q}_L = \dot{m}(h_1 - h_4) = \frac{6}{60} \times (246.36 - 88.82) = \boxed{15.754 \text{ kW}}$

But 1 ton refrigeration = 211 Kg/min = 200 Btu/min

$\dot{Q}_L = 6 \times (246.36 - 88.82) = \boxed{945.24 \text{ Kg/min}}$

$\Rightarrow \dot{Q}_L = \frac{945.24}{211} = \boxed{4.48 \text{ tons}}$

or $\dot{Q}_L = 4.48 \times 200$ Btu/min

$= 4.48 \times 200 \times 60$ Btu/h

$= \boxed{53760 \text{ Btu/h}}$

Problem III (20 points)

An ideal reheat Rankine cycle with water as the working fluid operates the inlet of the high-pressure turbine at 8000kPa and 450°C; the inlet of the low-pressure turbine at 500 kPa and 500°C; and the condenser at 10 kPa.

a) Sketch a T-s diagram of this cycle (**5 points**).

b) Determine the mass flow rate of water through the cycle if the net power output of the system is 5MW (**10 points**).

c) Determine the thermal efficiency of the cycle (**5 points**).

Solution:

• $h_1 = h_f @ 10\,kPa = \boxed{191.81\ KJ/Kg}$ and $v_1 = v_f @ 10\,kPa = 0.001010\ \frac{m^3}{Kg}$

• $h_2 = h_1 + v_1(P_2 - P_1)$

$= 191.81 + 0.001010(8000 - 10) = \boxed{199.88\ KJ/Kg}$

• @ ③: $P_3 = 8000\ KPa$, $T_3 = 450°C$ $\longrightarrow$ $\boxed{h_3 = 3273.3\ KJ/Kg}$, $S_3 = 6.5578\ KJ/Kg.K$

• @ ④: $P_4 = 500\ KPa$, $S_4 = S_3 = 6.5578\ KJ/Kg.K$ $\longrightarrow$ $S_f < S_4 < S_g$ ⇒ Sat. Liq. vap. Mixture

$$x_4 = \frac{S_4 - S_f}{S_{fg}} = \frac{6.5578 - 1.8604}{4.9603} = 0.947$$

$$\Rightarrow h_4 = h_f + x_4 h_{fg} = 640.09 + 0.947 \times 2108 = \boxed{2636.366\ KJ/Kg}$$

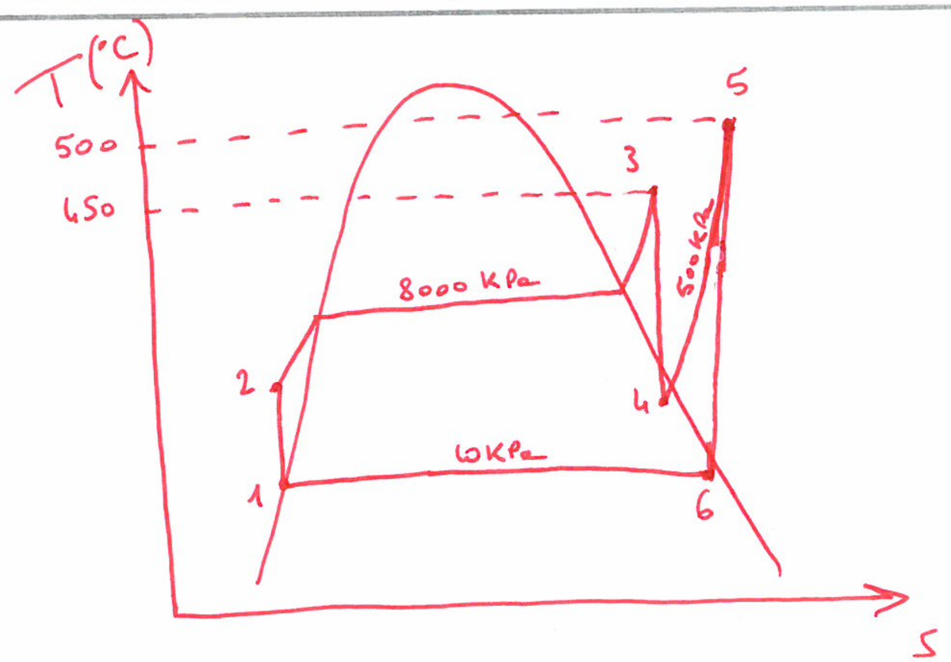
• @ ⑤: $P_5 = 500\ KPa$, $T_5 = 500°C$ $\longrightarrow$ $\boxed{h_5 = 3484.5\ KJ/Kg}$, $S_5 = 8.0893\ KJ/Kg.K$

@ ⑥: $P_6 = 10\ KPa$, $S_6 = S_5 = 8.0893\ KJ/Kg.K$ $\longrightarrow$ $S_f < S_6 < S_g$ ⇒ Sat. Liq. vap. Mixture

$$x_6 = \frac{S_6 - S_f}{S_{fg}} = \frac{8.0893 - 0.6492}{7.4996} = 0.992$$

$$\Rightarrow h_6 = h_f + x_6 h_{fg} = 191.81 + 0.992 \times 2392.1 = \boxed{2564.77\ KJ/Kg}$$

a)



b) $\dot{W}_{net,out} = 5\,MW = 5000\,KW$

but $\dot{W}_{net,out} = \dot{W}_{turb} - \dot{W}_{pump}$

$= \dot{W}_{t,I} + \dot{W}_{t,II} - \dot{W}_{pump}$

$= \dot{m}\left[(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)\right]$

$\rightarrow \dot{m} = \dfrac{\dot{W}_{net,out}}{h_3 - h_4 + h_5 - h_6 - h_2 + h_1} = \dfrac{5000}{3273.3 - 2636.366 + 3484.5 - 2564.77 - 199.88 + 191.81}$

$\boxed{\dot{m} = 3.23\ Kg/s}$

c) $\eta_{th} = \dfrac{\dot{W}_{net,out}}{\dot{Q}_{in}} = \dfrac{5000}{\dot{m}(h_3 - h_2 + h_5 - h_4)} = \dfrac{5000}{3.23(3273.3 - 199.88 + 3484.5 - 2636.366)}$

$\Rightarrow \boxed{\eta_{th} = 0.3947 \simeq 39.5\,\%}$

Problem IV (30 points)

Consider a two-stage cascade refrigeration system where both stages use refrigerant R-134a as the working fluid. The upper cycle operates between the pressure limits of 1.0MPa and 0.4MPa while the lower cycle operates between the pressure limits of 0.5MPa and 0.1MPa. If the mass flow rate of refrigerant through the upper cycle is 1kg/s and the isentropic efficiency of both compressors is 90%:

a) Sketch a T-s diagram of this cycle (**5 points**).

b) Determine the mass flow rate of refrigerant through the lower cycle (**10 points**).

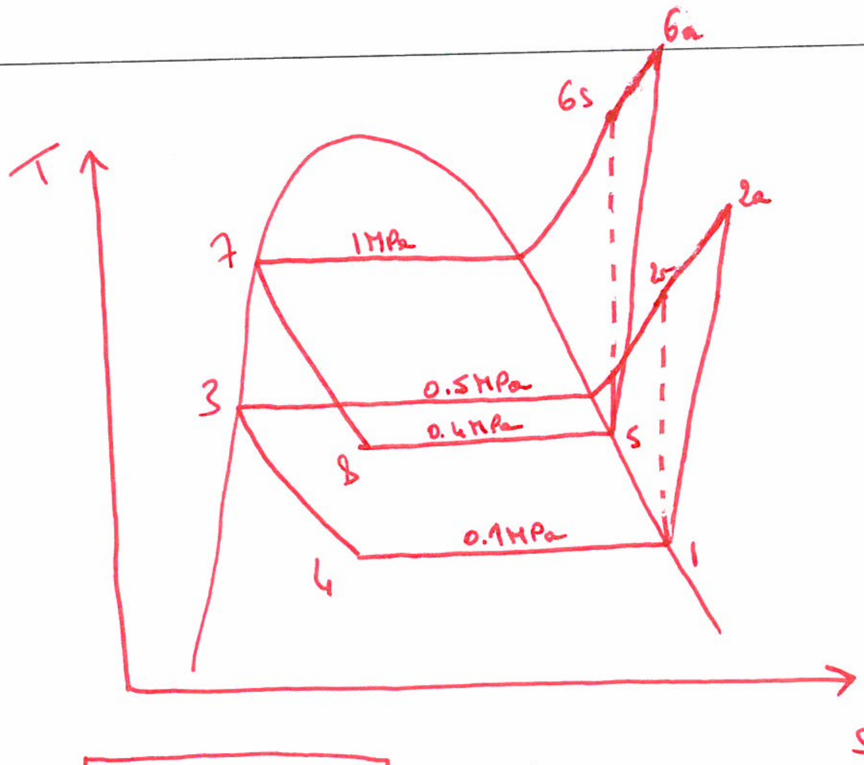
c) Determine the rate of heat removal from the refrigerated space (**5 points**).

d) Determine the total power input for the compressors (**5 points**).

e) Determine the Coefficient Of Performance of this refrigeration machine (**5 points**).

Solution:

a)



b) • $h_1 = h_g @ 0.1MPa = \boxed{234.44\ KJ/Kg}$ and $s_1 = s_g @ 0.1MPa = 0.95183\ KJ/Kg.K$

• @ (2s): $P_{2s} = 0.5MPa$, $s_{2s} = s_1 = 0.95183\ KJ/Kg.K$ $\Big\} \rightarrow h_{2s} = 267.5\ KJ/Kg$

but $\eta_c = 0.9 = \dfrac{w_s}{w_a} = \dfrac{h_{2s} - h_1}{h_{2a} - h_1}$

$$\rightarrow h_{2a} = \frac{h_{2s} - h_1}{0.9} + h_1 = \frac{267.5 - 234.44}{0.9} + 234.44 = \boxed{271.17\ KJ/Kg}$$

• $h_3 = h_f @ 0.5MPa = \boxed{73.33\ KJ/Kg}$

• $\boxed{h_4 = h_3 = 73.33\ KJ/Kg.}$

- $h_5 = h_g @ 0.4\text{MPa} = \boxed{255.55 \text{ KJ/Kg}}$ & $s_5 = s_g @ 0.4\text{MPa} = 0.92691 \text{ KJ/Kg.K}$

- @ (6s), $P_{6s} = 1\text{MPa}$, $s_{6s} = s_5 = 0.92691 \text{ KJ/Kg.K}$ $\rightarrow h_{6s} = 274.58 \text{ KJ/Kg}$

and $h_{6a} = \dfrac{h_{6s} - h_5}{0.9} + h_5 = \dfrac{274.58 - 255.55}{0.9} + 255.55 = \boxed{276.69 \text{ KJ/Kg}}$

- $h_7 = h_f @ 1\text{MPa} = \boxed{107.32 \text{ KJ/Kg}}$
- $h_8 = h_7 = \boxed{107.32 \text{ KJ/Kg}}$

Energy balance on heat exchanger between the two cycles:

$\dot{m}_{upper}(h_5 - h_8) = \dot{m}_{lower}(h_{2a} - h_3)$

$\Rightarrow \dot{m}_{lower} = \dot{m}_{upper} \dfrac{h_5 - h_8}{h_{2a} - h_3} = 1 \times \dfrac{255.55 - 107.32}{271.17 - 73.33} = \boxed{0.74924 \text{ Kg/s}}$

c) $\dot{Q}_L = \dot{m}_{lower}(h_1 - h_4) = 0.74924 \times (234.44 - 73.33) = \boxed{120.71 \text{ KW}}$

d) $\dot{W}_{comp} = \dot{W}_{comp1} + \dot{W}_{comp2}$

$= \dot{m}_{lower}(h_{2a} - h_1) + \dot{m}_{upper}(h_{6a} - h_5)$

$= 0.74924(271.17 - 234.44) + 1 \times (276.69 - 255.55)$

$= \boxed{48.66 \text{ KW}}$

e) $COP = \dfrac{\dot{Q}_L}{\dot{W}_{comp,i}} = \dfrac{120.71}{48.66} = \boxed{2.48}$